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STANDARD BAYES ESTIMATION WITH EXTENSION LOOS FUNCTION FOR PARAMETER WEIBULL DISTRIBUTION

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ABSTRACT

The objective of this study is to introduce three estimators of parameter Weibull distribution, depending on standard Bayes estimation and modified Bayes estimation. Then, using a simulation study (MATLAB program), to find the best one base on MPE and MSE.

KEYWORDS: Bayes Estimation, Jeffery Prior Information, Simulation Study

INTRODUCTION

The theorem of Bayes provides a solution on how to learn from data. Related to survival function and by using Bayes estimator, Elli and Rao (1986), estimated the shape and scale parameters of the Weibull distribution, by assuming a weighted squared error loss function. They minimized the corresponding expected loss, with respect to a given posterior distribution. Sinha & Sloan (1988) obtained, Bayes estimator of three parameters Weibull distribution and compared the posterior standard deviation estimates, with the corresponding asymptotic standard deviation of their maximum likelihood counterparts and numerical examples are given. In 2002, Klaus Felsenstein developed Bayesian procedures, for vague data. These data were assumed to be vague in the sense that the likelihood is a mixture of the model distribution with error distribution. An extension of Jeffery prior information with square error, loss function in exponential distribution was studied by Al-Kutubi (2005). In this paper, Alkutubi (2009) proposed an extension of Jeffery prior information, with a new loss function and then compare it with standard Bayes, to find the best. In this paper, we will introduce three estimators to the parameters Weibull distribution, using standard Bayes estimation and extension loss function. We will then compare between them by a simulation study, to find the best one base on MPE and MSE.

MATERIALS AND METHODS

Let $t_1, t_2, \ldots, t_n$ be the life time of a random sample of size n with distribution function and probability density function. In the Weibull case, we assumed that the probability density function of the lifetime is given by

$$(t, \theta, c) = \frac{c}{\theta}\left(\frac{t_i}{\theta}\right)^{c-1} \exp\left(\frac{-t}{\theta}\right)^{c}$$

To obtain Bayes estimator, the following steps are needed [2], [3].

A number of n items put to the test and the lifetimes of this random samples are recorded with the probability density function $f(t, \theta, c)$.The life time ability density function $f(t, \theta, c)$ are regarded as a conditional probability density function $f(t|\theta, c)$, where the marginal probability density function of θ is given by $g(\theta)$, the Jeffery prior information.

$$f(t, \theta, c) = \frac{c}{\theta}\left(\frac{t}{\theta}\right)^{c-1} e^{\left(-\frac{t}{\theta}\right)^{c}}$$

$\frac{\partial \ln f(t,\theta,c)}{\partial \theta^2} = \frac{c}{\theta^2} - 2c\left(\frac{t}{\theta}\right)^{c-1}\left(\frac{t}{\theta^3}\right) - c(c-1)\left(\frac{t}{\theta}\right)^{c-2}\left(\frac{t}{\theta}\right)^2$=M

We find Jeffery prior by taking $g(\theta) \propto \sqrt{I(\theta)}$, where fisher information $I(\theta)$ is given by

$$I(\theta) = -n\, E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial \theta^2}\right)$$

$$E\left(\frac{\partial^2 \ln f(t,\theta,c)}{\partial \theta^2}\right) = E(M) = \int_0^\infty M\, \frac{c}{\theta}\left(\frac{t}{\theta}\right)^{c-1} e^{-\left(\frac{t}{\theta}\right)^c} dt = \frac{-c^2}{\theta^2}$$

Then $I(\theta) = \frac{nc^2}{\theta^2}$, so the Jeffery prior information is given by $g(\theta) = k\sqrt{I(\theta)} = k\sqrt{\frac{nc^2}{\theta^2}} = k\frac{c\sqrt{n}}{\theta}$

The joint probability density function is:

$$= \frac{kc^2\sqrt{n}}{\theta^{n+1}}\left(\frac{\sum t_i}{\theta}\right)^{c-1} exp\left(-\left(\frac{\sum t_i}{\theta}\right)^c\right)$$

The marginal probability density function of $(t_i, \dots t_n, \theta, c)$ is given by:$= \frac{kc\sqrt{n}}{(\sum t_i)^n}\left(\frac{n-1}{c}\right)!$

Then posterior distribution is,

$$= \frac{c(\sum t_i)^n\left(\frac{\sum t_i}{\theta}\right)^{c-1}\exp\left(-\left(\frac{\sum t_i}{\theta}\right)^c\right)}{\theta^{n+1}\left(\frac{n-1}{c}\right)!}$$

By using squared error loss function $\ell(\hat{\theta}-\theta) = c(\hat{\theta}-\theta)^2$, we can obtain the Risk function, such that

$$R(\hat{\theta},\theta) = EL(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \pi(\theta|t_1, \dots t_n)d\theta$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, then Bayes estimator is $\widehat{\theta_1} = \frac{\sum t_i}{\left(\frac{n-1}{c}\right)}$

By using new extension loss function [1], we can get new estimator of parameter Weibull distribution, such that

$R(\hat{\theta},\theta) = \int_0^\infty \theta^c(\hat{\theta}-\theta)^2\, \pi(\theta|t_1, \dots t_n)d\theta$ Solving equation in above to get the second Bayes estimator

$$\hat{\theta}_2 = \frac{\sum t_i}{\left(\frac{2n}{c}\right)}$$

By c=2, The another new estimator of parameter Weibull distribution, such that

$R(\hat{\theta},\theta) = \int_0^\infty \theta^2(\hat{\theta}-\theta)^2\, \pi(\theta|t_1, \dots t_n)d\theta$ Solving the above equation to get the second Bayes estimator

$$\hat{\theta}_3 = \frac{\sum t_i}{\left(\frac{n-2}{c}\right)}$$

RESULTS

In a simulation study, we have chosen n=50, 75, 100 to represent small, moderate and large sample size, several values of parameter θ=0.5, 1, 1.5, and values of the new loss function c = 0.4, 0.8. The number of replications used was R=1000. The simulation program was written by using the Matlab program. After the parameter was estimated, mean square error (MSE) and mean percentage error (MPE) were calculated, to compare the methods of estimation, where

$$\text{MSE}\left(\hat{\theta}\right)=\frac{\sum_{i=1}^{1000}\left(\hat{\theta}-\theta\right)^2}{R} \text{ and } \text{MPE}\left(\hat{\theta}\right)=\frac{\left[\sum_{i=1}^{1000}\frac{\left|\hat{\theta}_i-\theta\right|}{\theta}\right]}{R}$$

The results of the simulation study are summarized and tabulated in Table 1 and Table 2 for the MSE and the MPE of the three estimators for all sample sizes and θ values respectively. It is obvious from these tables, a Bayes estimator with the Jeffery prior information, $\widehat{\theta_1}$is the best estimator. But $\widehat{\theta_3}$ in most cases have the largest MSE and MPE

Table 1: The Ordering of the Estimators with Respect to MSE

Size	θ	C	Theta Hat		
			$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
50	0.5	0.4	0.100	0.135	0.211
		0.8	0.121	0.133	0.211
	1	0.4	0.182	0.209	0.229
		0.8	0.189	0.209	0.229
	1.5	0.4	0.214	0.231	0.233
		0.8	0.223	0.235	0.233
75	0.5	0.4	0.125	0.131	0.139
		0.8	0.125	0.132	0.132
	1	0.4	0.121	0.122	0.122
		0.8	0.123	0.123	0.123
	1.5	0.4	0.122	0.125	0.130
		0.8	0.126	0.126	0.133
100	0.5	0.4	0.091	0.093	0.094
		0.8	0.091	0.093	0.094
	1	0.4	0.088	0.089	0.089
		0.8	0.088	0.089	0.089
	1.5	0.4	0.088	0.088	0.089
		0.8	0.088	0.088	0.090

Table 2: The Ordering of the Estimators with Respect to MPE

Size	θ	C	Theta Hat		
			$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
50	0.5	0.4	0.010	0.013	0.021
		0.8	0.010	0.013	0.021
	1	0.4	0.012	0.020	0.022
		0.8	0.012	0.020	0.022
	1.5	0.4	0.013	0.013	0.024
		0.8	0.013	0.013	0.024
5	0.5	0.4	0.012	0.012	0.013
		0.8	0.012	0.012	0.014
	1	0.4	0.011	0.013	0.014
		0.8	0.011	0.013	0.014
	1.5	0.4	0.010	0.013	0.013
		0.8	0.010	0.013	0.012
100	0.5	0.4	0.006	0.007	0.009
		0.8	0.006	0.007	0.009
	1	0.4	0.005	0.006	0.007
		0.8	0.004	0.004	0.007
	1.5	0.4	0.002	0.003	0.005
		0.8	0.002	0.003	0.005

DISCUSSION

The results of the simulation study are summarized and tabulated in Table 1 and Table 2 for the MSE and the MPE of the three estimators for all sample sizes, $\theta$and C values respectively. The order of the estimator is from the best (smallest MSE) to the worst (largest MSE). It is obvious from these tables, a Bayes estimator with Jeffery prior information and standard loss function, $\hat{\theta}_1$ is the best estimator. In most of the cases, it is apparent that, the Bayes estimator with Jeffery prior information and extension new loss function $\hat{\theta}_2$ is the next best estimator. But, standard Bayes estimator with new loss function $\hat{\theta}_3$ in most cases has the largest MSE and MPE.

CONCLUSIONS

The Bayes estimator, with Jeffery prior information $\hat{\theta}_1$ is the best estimator, when compared it with the other estimators. We can easily conclude that, MSE and MPE of Bayes estimators decrease with an increase of sample size.

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